Title For Title

Author 1[1,*] and Author 2[2,3,†]

[1] *Author 1 address*
[2] *Author 2 address*
[3] *address 3*

(Dated: November 3, 2020)

Abstract

In recent years, moiré superlattices have been studied extensively in various 2D van der Waals heterostructures exemplified by graphene and transition metal dichalcogenide (TMD) multilayers [1–3]. These moiré systems exhibit a variety of remarkable electronic properties due to strong correlation effects in flat minibands. Besides graphene and TMD, another large family of moiré superlattices can be found in topological insulators [4–14], in particular Bi_2Se_3 and Bi_2Te_3. When these bulk crystals are grown by the molecular-beam epitaxy (MBE), it is common to find a small rotational misalignment of topmost quintuple layers, leading to a moiré superlattice on the surface. Interestingly, a scanning tunneling microscope (STM) measurement [6] has directly observed such moiré superlattice in Bi_2Te_3 and found multiple sharp peaks in the local density of states (LDOS). Despite the ubiquity of moiré superlattices in TI, their effects on topological surface states have not been studied theoretically.

In this letter, we study moiré surface states of TI. The topological nature of TI surface states prevents them from gap opening as long as time-reversal symmetry is preserved, hence the moiré surface states do not form isolated mini bands, unlike other moiré systems such as graphene and TMD. Instead, we find prominent van Hove singularities (VHS) in moiré surface states which give rise to divergent density of states (DOS). Under appropriate conditions, some of these VHS exhibit power-law divergent DOS, which are known as *high-order* VHS [15].

We further study superconductivity at high-order VHS, where the electron-phonon interaction effect is significantly enhanced due to the divergent DOS. We find a new analytic formula for the superconducting critical temperature T_c (see Eq. (**??**)), which exhibits a *power-law* dependence of the retarded electron-phonon interaction λ^* and is thus parametrically enhanced with respect to the exponentially small T_c in ordinary metals and at ordinary VHS [16, 17].

This work is organized as follows: we first study Dirac fermions in the periodic potential as a model of moiré topological insulator surface states. Then, we solve the gap equation for the superconducting critical temperature T_c near a general high-order VHS taking account of electron-electron repulsion within the Anderson-Morel approximation [18]. In the end, we discuss several experiment platforms to realize our model of Dirac fermions.

Dirac fermions in periodic potential.

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* Author1 e-mail
† Author2 e-mail

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Supplemental Material Title

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$$H = E\Psi \tag{S1}$$